A sequential procurement model for a PPP project pipeline

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Abstract

Public-private partnerships have seen the daylight in response to the adagio that project responsibilities and risks should be efficiently allocated between the public and the private sector. Nevertheless, the considerable bidding costs inhibit the competition in the market. A trustworthy project agenda, also called a project pipeline, could substantiate the PPP market's attractiveness in the belief that a current success results in a knowledge and cost advantage in future tenders. This paper builds a sequential PPP procurement model and heuristically approximates the Markov perfect equilibrium in which contractors determine how much money they are willing to invest in the bid preparation and which mark-up is appropriate for each project in the pipeline. A pipeline of projects pushes down the mark-ups and the government procurement cost. Nonetheless, according to the experiments, it are only players with an initial experiential advantage who tend to make higher investment efforts so that additional governmental policies, like a fractional reimbursement of the bid preparation efforts, might be required to level the playing field.

Keywords: public-private partnerships, sequential procurement auction, Markov perfect equilibrium, best response heuristic

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1. Introduction

This paper offers a theoretical approach in analyzing public-private partnership (PPP) procurement for a sequence of projects. Since risk assessment and mitigation strategies continue to receive a great emphasis within the project management field, a PPP is a recent phenomenon that aims to attain an efficient risk allocation between a public sector entity and a private consortium. The latter party may, in addition to the construction, also engage in the design, maintenance and/or operation of the project. Due to the complex project nature and the high societal value that is at stake, financial performance requirements and high quality standards are important challenges one needs to face. Therefore, contractors ought to prepare a qualitative bid proposal to submit to the contracting government. The preparation of this proposal is costly (e.g., consultancy cost, working cost, design cost) and the risk of not being awarded the contract is empirically claimed to be a burden for contractors [1, 7, 25]. In order to ensure the competition, policy makers are endowed to seek for feasible ways to substantiate the PPP market's attractiveness. This paper investigates whether a project pipeline succeeds in positively affecting the contractors' bidding behavior. This paper defines the project pipeline as follows: "A PPP project pipeline is a sequence of similar projects that the government ensures to tender in the near future. It may concern totally independent projects or sub-projects that serve a larger purpose". If the government agrees on a clear project agenda, contractors might be more eager to enter a particular market. Winning a contract might result in a competitive advantage in future tenders and, moreover, past losses can be recovered in future tenders.

This paper converts the PPP procurement setting into a sequential procurement auction format in which the government acts as the decision maker. For each project in the pipeline, the contractors or consortia that are invited for the tender determine their appropriate bid preparation effort willingness (i.e., the investment decision) and the mark-up percentage (i.e., the mark-up decision). Within this competitive framework, the bidders are heterogeneous in their experience levels, because winning a contract increases the experience in future tenders. The past experience a bidder has obtained together with the pre-tender investment efforts could lead to an increased ability to estimate the project cost on the one hand and to efficiency gains that result in a lower expected cost on the other hand. The study analytically characterizes the equilibrium of this stochastic game and employs an experimental setting to approximate the Markov perfect equilibrium for different project cost features and a varying number of projects in the pipeline.

The method applies a best response heuristic for which an algorithmic approximation that is based on both an electromagnetism-like mechanism as well as a local search procedure efficiently detects the best response. This approach succeeds in looking at the entire, continuous action search space in order to identify candidate equilibria.

This research benefits practitioners and policy makers by offering a theoretical foundation for PPP procurement modeling. Next to its practical relevance, this work contributes to different stretches in the academic literature. First of all, competitive procurement models are scarce in the PPP literature while they could gain insights into a field in which empirical bidding data are often inadequately available. Moreover, theoretical models allow for an initial feasibility assessment of policy measures. Secondly, the PPP framework offers research opportunities within the OR community. The complex bidding framework is related to the traditional procurement auction field. In this vein, the paper's model can be considered as an auction format with asymmetries among the bidders with respect to the project cost distribution and the option to invest in information acquisition. The pipeline concept adds a synergetic, sequential feature in the sense that past successes lead to a knowledge and cost advantage in future tenders.

The remainder of the paper is structured as follows. Section 2 explores the related operations research literature. Section 3 delves into the analytical foundation and the Markov equilibrium concept and subsequently discusses the experimental implementation and the equilibrium approximation algorithm. The outcomes of the scenario analysis are covered in Section 4 and are taken forward into a concluding discussion in Section 5.

2. Literature

The study of the sequential PPP procurement format is closely linked to the sequential auction theory where multiple units are auctioned in a strictly sequential fashion and where the winner is considered to be the highest bidder. The views on the expected price trends vary. Weber [47] proved that in the case of an uncertain common value, bids follow a Martingale, while the majority of theoretical studies argue that bid prices are declining. Examples include Von der Fehr [45] who studies auctions with participation cost and Kannan [20] in a setting with complete bid revelation. Branco [5] supports the price decline in the case of complementarities between identical objects, which is nuanced by Sørensen [40] in the case of a large number of stochastically equivalent objects or when probabilities to draw a large value are high. Engelbrecht-Wiggans [12] attributes this to the trade-off between a reduced competition effect and a second effect that is related to the number of chances to win. In addition, Menezes and Monteiro [27] relate the price trend to the type of synergies of objects. Hence, the sequential auction literature can be classified according to this object-related dimension. The objects may be complements and create synergies which has been extensively studied by Branco [5], Menezes and Monteiro [27] and De Silva et al. [10]. Alternatively, the objects can be homogeneous in nature in the sense that they are perfect substitutes [22, 52]. Finally, the objects may have the same stochastic feature and are said to be stochastically equivalent [12, 36-37]. Our model assumes that the projects have a constant stochastic nature, but contractors' experience in past projects has an influence on the cost probability distribution of future projects. Apart from the type of the tendered objects, modeling approaches also differ in their consideration of the capacity constraints. Whereas Katzman [22] and Katehakis and Puranam [21] for instance do not include a capacity constraint, the contributions of Engelbrecht-Wiggans [12], Milgrom and Weber [29], Elmaghraby [11] and Reiβ and Schöndube [36] assume a single-unit demand. Other studies include monetary constraints [35] or restrict the availability of man-hours [42]. The PPP model does not account for capacity constraints and the results point towards rising mark-ups in later stages of the pipeline. Next to the mark-ups, contractors in the PPP model also make a decision on the pre-tender research efforts, but to the best of our knowledge, there have been no previous studies on the impact of a sequential mechanism on the willingness for information acquisition. The work of for instance Persico [34], Bergemann and Välimäki [3] and Shi [39] considers a single-shot game in which the bidders can gain information on the value of the auctioned object, but they do not extrapolate to a multi-unit environment.

More particularly, our paper is closely related to the contribution of Takano et al. [42] who study the competitive bidding strategy in a sequential setting with inaccurate cost estimates. Building upon earlier research from Naert and Weverberg [31] and King and Mercer [24], the authors explicitly account for the fact that bids are usually correlated with the estimated cost that is subject to inaccuracies. Takano et al. [42] apply a scenario-based approach for the cost estimates and a capacity constraint. Our work differs from and extends this study in several aspects. First of all, in addition to the mark-up choice, a bidder makes an investment decision that reduces the uncertainty and the expected cost. Secondly, Takano et al. [42] simulate costs and bids for the competitors, while we look at action equilibria by simultaneously optimizing each bidders' pay-off. Thirdly, our model does not include capacity constraints or a value at risk constraint and deals with a constant number of heterogeneous contractors. Furthermore, it builds on the single-shot PPP procurement model of De Clerck and Demeulemeester [9] in three ways. Firstly, this paper extends the scope towards a multi-project environment. Besides, the current study considers a continuous strategy space and last but not least, the contractors' pay-offs are not derived by a multi-agent simulation approach, but are exactly computed for presumed distributions.

The methodological approach is based on a dynamic programming model for which Markov equilibria are approximated based on a best response heuristic. The Markov perfect equilibrium concept, drawn from the study of Maskin and Tirole [26], is a solution concept that solves the stochastic game with a finite number of stages and where the pay-off and probabilistic transitions depend on the current state and the chosen actions [38]. The solution concept is not uncommon within the auction literature. In a sequential auction setting with randomly arriving bidders, Said [37] derives Markov equilibria to discuss bid shading (i.e., placing a bid below the estimated value) as a consequence of an option value of participation in future auctions. Within a procurement setting, Katehakis and Puranam [21] study the cost minimization objective of a buyer who wants to procure a fixed number of products and Yildirim [49] considers the optimal mechanism for piecewise procurement of large-scale projects. While these studies and our paper do not account for capacity constraints, Jofre-Bonet and Pesendorfer [18] include capacity limitations when defining the states of the game and they use empirical highway procurement bidding data to develop an estimation method for a repeated auction. Whereas most models allow for a different number of bidders in each stage, the PPP model assumes repeated competition among a constant set of bidders (like in [14] and [29]) and includes the current levels of the bidders' past experience and the number of projects remaining in the state variable. Hence, the PPP model allows for heterogeneity among the bidders. The theoretical model of Jofre-Bonet and Pesendorfer [19] and the empirical studies by De Silva et al. [10] and Wolfram [48] account for endogenously appearing asymmetries due to synergies, but in the PPP setting bidders may ex ante differ in the cost probability distributions which is also the case in the work of Reiβ and Schöndube [36]. Moreover, in contrast to models that study infinite timeframes (e.g., [16, 33, 37, 51]), the PPP pipeline has a finite nature which is a logical consequence of the magnitude of the projects and the limited budget horizon of governments.

The experimental set-up relies on a heuristic approach to derive the equilibrium. Algorithmic game theory attempts to deal with the complexity of real-life models [32]. Algorithms that use best response reasoning have been successfully implemented for instance for empirical games, routing games or dynamic oligopoly models and often limit the search space and the number of computationally expensive pay-off calculations (e.g., [13, 41, 46]). Nevertheless, a best response heuristic does not always converge and, to the best of our knowledge, formal proofs of convergence are limited to super-modular games with unique Nash equilibria [28] and congestion games [30]. The experimental results in this paper focus on the scenarios for which convergence has occurred.

Next to its contribution to the operations research field, this paper adds value to the PPP project management literature. The number of papers on PPPs has been proliferating and an overview of topics can be found in Ke et al. [23] and Tang et al. [43]. Past experiences have led to concise lists of success factors and key performance indicators (e.g., [17, 50]). Nevertheless, the majority of the studies are limited to attaining single-project success without looking at the broader PPP picture. However, empirical results also underline long-term and country-specific factors like the country's government reputation, legal framework and economic stability [2, 8, 50] or the importance of PPP units in promoting PPPs [44]. In this vein, a consultancy report from KPMG [25] argues for efforts to levy barriers to competition and procurement inefficiencies. The report suggests that the pipeline concept could be beneficial, but this has not been covered within the academic PPP literature. Additionally, modeling the expensive bidding process has not received substantial consideration. In a single-project setting, Ho [15] included government reimbursements to incentivize bidders to invest in more qualitative bids into a game-theoretical PPP bidding model. De Clerck and Demeulemeester [9] relate to Ho's work, but include project contingencies and heterogeneous bidders into their bidding model. The authors claim that a bid cost reimbursement could be an effective tool to increase competition.

3. Methodology

3.1 Competitive bidding procedure

This paper extends the work of De Clerck and Demeulemeester [9] in which a single-project environment has been analyzed. The analysis of competitive bidding for PPP projects resulted in a complex procurement auction environment. In a stage z, contractors that are invited for the tender will first determine how much effort they are willing to put into the bid preparation. This investment may result in a reduction of the cost uncertainty as well as in a cost advantage. Moreover, the heterogeneity among contractors leads to more advantageous cost probability distribution functions for more experienced players, which means a smaller variance and a lower average expected cost. After the investment decision, each contractor p estimates the project cost which is subject to estimation errors. Next, the mark-up is determined and applied to the estimated cost, resulting in the bid for project z. The lowest bidding contractor is granted the project. Afterwards, the tendering procedure for stage $z + 1$ is initiated. The sequential model adds an additional feature: winning a project in the sequence results in additional experience for all subsequent projects.

3.2 Sequential bidding model

Given is a commonly known project sequence $\mathcal{Z} \coloneqq \{1,2,\dots,Z\}$. We want to identify the strategy equilibrium of the subgame $e = (e_1, e_2, \dots, e_P)$ that defines the initial experience setting of the players of the game. This setting is a stochastic game, as has been introduced by Shapley [38]. A stochastic game is a finite or infinite dynamic game that is played by one or more players with probabilistic transitions between a finite number of states. In this setting, the players are assumed to be long-lived and to have unlimited capacity to perform all the projects of the pipeline. In each stage of the sequential game, the contractors want to optimize their total expected pay-off which consists of the instantaneous pay-off of the current stage and an expected continuation value of the pay-offs in future stages. Fig. 1 serves as an example of an (optimal) sequential strategy in a three-project game tree and also clarifies the notation of this section. The figure’s notation uses an asterisk with the actions to indicate that the reported strategy is an optimal strategy in the example. We are looking at Markov strategies and identify a Markov perfect equilibrium (MPE) as presented by Maskin and Tirole [26]. This is justified as the current play is only influenced by the expected pay-offs of future projects on the one hand and the state variable on the other hand. But before moving to the equilibrium determination in Section 3.4, this section explains the generic structure of the different elements of the total expected pay-off, while Section 3.3 elaborates on the specific implementation for the PPP setting of the paper.

For a given project or stage $z \in \mathcal{Z}$ from the sequence, we determine the current state $\theta^z \in \Theta^z$ as the combination of the current experience levels and the number of remaining projects in the sequence $Z - z$ or $\theta^z = (e, Z - z) = (e_1, e_2, \dots, e_P, Z - z)$. The state variable θ^z in fact represents the history of what has happened earlier in the stochastic game. Consequently, we assume that past investment and mark-up decisions solely impact the current behavior by having won or lost the tender. For a given state θ^z, a set of actions $\mathcal{A}_p^z = \mathcal{A}_p^z(\theta^z)$ is available for each player. We assume that the set of available actions is the same in every state of the game, so that we can refer to the set of actions as $\mathcal{A}_p$. An action a_p^z in a given state θ^z is composed of two elements: the amount of pre-tender investment $i\left(a_p^z\middle|\theta^z\right)$ a player p is willing to adopt and which is expressed as a percentage of an initial cost base equal to 1 and the mark-up $m\left(a_p^z\middle|\theta^z\right)$ defined as a percentage that is applied to the estimated project cost. The vector $a^z = (a_1^z, \dots, a_P^z)$ represents the action profile or the combination of actions (i.e., the investment and mark-up percentages) for all P players. If we define the set of players as $\mathcal{P}$ and the set of action profiles as $\mathcal{A} = \times_{p\in\mathcal{P}} \mathcal{A}_p$, then we determine the transition probabilities $\mathcal{Q}$ from $\mathcal{A} \times \Theta^z$ to Θ^{z+1} so that $\mathcal{Q}(\theta^{z+1}|a^z, \theta^z)$ represents the probability of arriving in state θ^{z+1} from the current state θ^z with an action profile a^z. For each stage z, the vector $\pi^z(a^z|\theta^z)$ of dimension $\mathbb{R}^P$ represents the total expected pay-off for each of the players, with θ^z the given state vector in this node of the stochastic game and a^z the decision variables or the action profile of the players. Each player-specific element $\pi_p^z(a^z|\theta^z)$ of this vector is the sum of an instantaneous pay-off $\rho_p^z(a^z|\theta^z)$ and the player's expected value $\mathcal{V}_p^{z+1}(a^z|\theta^z)$ for the future stages $\{z+1, \dots, Z\}$ discounted with a factor Δ $(0 < \Delta \leq 1)$. This continuation value depends on the actions that are taken at the current state, because these actions will determine the probability of arriving in each of the states in stage $z+1$. These probabilities are represented by a transition matrix $\mathcal{Q}$ that represents, for a given action profile a^z and a given state θ^z, the probabilities of arriving in all states θ^{z+1}. In summary, the player-specific expected pay-off function that player p wants to optimize at stage z equals:

$$\pi_p^z(a^z|\theta^z) = \rho_p^z(a^z|\theta^z) + \Delta\mathcal{V}_p^{z+1}(a^z|\theta^z)$$

with

$$\mathcal{V}_p^{z+1}(a^z|\theta^z) = \sum_{\theta^{z+1}\in\Theta^{z+1}} \mathcal{Q}(\theta^{z+1}|a^z, \theta^z)\pi_p^{z+1}(a^{z+1}|\theta^{z+1})$$

and for each $z \in \mathcal{Z}$ and $\theta^z \in \Theta^z$. Furthermore, we assume that the continuation value of the final project of the pipeline is zero, or $\mathcal{V}_p^{Z+1}(a^Z|\theta^Z) = 0$ for all p. Hence, the total expected pay-off is a linear combination of instantaneous pay-offs. Before moving on to the characterization of the equilibrium, the instantaneous pay-off for the PPP model needs to be determined.

[FIGURE 1 TO BE INSERTED HERE]

3.3 Expected pay-off calculation in the PPP model

First of all, we assume that the state of the game, which consists of the experience levels and the remaining projects in the pipeline, is common knowledge to all players. Moreover, the general risk structure of the projects is the same for all the projects in the pipeline. That means that, without considering experience and pre-tender investment, the ex ante cost probability distribution and its parameters do not change. The experience and the current investment level may change the shape of the cost probability distribution. The model only incurs project-specific investment efforts, so that investments only directly affect the current stage and do not contribute to the knowledge base of future stages. The experience as well as the investment have an impact on the expected value and the variance of the cost probability distribution. A user-defined experience scale [0,10] is defined with e_u the number of intervals on the scale. Equivalently, e_u refers to the number of projects won beyond which no extra experience can be adopted. Consequently, the conversion of experience level e_p of player p on the experience scale [0,10] is obtained by: $10 * \frac{e_p}{e_u} = e_p^c$. If $e_u = 2$, an inexperienced player has $e_p = e_p^c = 0$ and a medium experienced and a highly experienced player have levels $e_p = 1$ ($e_p^c = 5$) and $e_p = 2$ ($e_p^c = 10$) respectively. The impact of experience and investment is implemented with diminishing scale effects and moreover, Gaussian distributions are assumed. Consequently, the contractor-specific project cost probability distribution $c_p^z(a_p^z|\theta^z)$ for the player's action a_p and given the state variable θ^z is of the form $N\left(1 + g_p(a_p^z|\theta^z), \sigma_p^2(a_p^z|\theta^z)\right)$. We state that $g_p(.)$ represents the fraction of the cost that is accountable for the lack of experience or investment. We assume that a less experienced player and a player who does not invest, will, on average, execute the project at a higher cost. In order to account for diminishing scale effects, the model defines $g_p(a_p^z|\,\theta^z) = \beta_i e^{-\mu_i\left(100 i(a_p^z|\theta^z)\right)} + \beta_e e^{-\mu_e e_p^c(\theta^z)}$ which means that the expected value of the contractor-specific cost distribution at a particular state of the game is dependent on the amount of investment $i(.|.)$ and the (converted) experience level $e_p^c(.)$ in state θ^z. The parameters β_i and β_e reflect the maximum cost impact for players who do not invest and players without experience, respectively. μ_i and μ_e represent the associated growth parameters.

Moreover, the variance of the distribution is dependent on the experience level and the investment. Implementation-wise, we define $\sigma_p^2\left(a_p^z|\theta^z\right) = \sigma^2 + \left(\gamma_i e^{-\lambda_i\left(100i\left(a_p^z|\theta^z\right)\right)}\right)^2 + \left(\gamma_e e^{-\lambda_e e_p^c(\theta^z)}\right)^2$. The total variance of the cost distribution is composed of an uncontrollable part σ^2 that is equivalent for all players, a part attributed to the (lack of) investment (i.e., the second term) and a fraction related to the (lack of) experience (i.e., the third term). Furthermore, γ_i represents the share of the variance or risk that is caused by a lack of investment and γ_e is related to the knowledge impact of experience and the associated growth parameters are λ_i and λ_e. The composition of this variance function relates to the philosophy of diminishing scale effects for each of the terms and to the fact that we assume that the total variance is the sum of the three distinct variance relationships.

$\left(1 + g_p(.)\right)$ will be the actual cost if player p wins the bid. However, the player does not know this yet at the time of bidding. Instead, he receives a signal which is a bid generated from $c_p\left(a_p^z\middle|\theta^z\right)$. In order to account for the inaccuracy of this signal, the bidder sets a mark-up $m(a_p^z|\theta^z)$ that represents the risk premium and the profit margin.

Assume, for instance that $e_u = 2$, $\sigma = 0.05$, $\gamma_i = \gamma_e = \beta_i = \beta_e = 0.1$, $\lambda_e = \mu_e = \lambda_i = \mu_i = 0.25$, then a player with no experience and a 1% investment choice has a cost probability distributed as $N(1.17788{,}0.01856)$ and for a player with experience level 1 ($e_p^c = 5$) and a 2% investment choice, we get $N(1.08930{,}0.00699)$. When a contractor p estimates the project cost $\widetilde{E_p^z}$, which is generated from the cost probability distribution $c_p^z(.)$, he applies a mark-up to the expected cost to arrive at the bid $\left(1 + m\left(a_p^z\middle|\theta^z\right)\right)\widetilde{E_p^z}$. Consequently, the bid probability distribution given a player's action a_p^z and a state θ^z equals $b_p^z\left(a_p^z\middle|\theta^z\right)$ and is characterized by the normal distribution $N\left(\left(1 + m\left(a_p^z\middle|\theta^z\right)\right)\left(1 + g_p\left(a_p^z\middle|\theta^z\right)\right), \left(1 + m\left(a_p^z\middle|\theta^z\right)\right)^2 \sigma_p^2\left(a_p^z\middle|\theta^z\right)\right)$ with the associated cumulative bid probability distribution B_p^z. It is assumed that the lowest bidding contractor is granted the project, so the probability of winning contract z with P players is $q_p^z(a^z|\theta^z) = \int_{-\infty}^{+\infty} b_p^z\left(x_p\right)\left[\prod_{k\in\mathcal{P}\setminus\{p\}}\left(1 - B_k^z(x_k)\right)\right]dx_p$. These probabilities also refer to the transition probabilities $Q(\theta^{z+1}|a^z, \theta^z)$ to move from a state θ^z to θ^{z+1}. The contractor that wins the project receives the proposed bid and pays the actual cost $\left(1 + g_p(.)\right)$ of the project and the monetary investment effort. Additionally, this player obtains an updated experience level in all future stages of the game, i.e., until $e_p = e_u$. The pay-off of the other contractors is determined by the lost investment. Nevertheless, the government might reimburse losing bidders for the investment efforts with a fraction d (Section 4.6). Consequently, the instantaneous expected pay-off for a player p in an action profile a^z and state θ^z is given by: $\rho_p^z(a^z|\theta_z) = q_p^z(a^z|\theta^z)\left(E\left[\widetilde{B_p^z}\middle|p \text{ has won}\right] - \left(1 + g_p\left(a_p^z|\theta^z\right)\right) - i\left(a_p^z\middle|\theta^z\right)\right) - \left(1 - q_p^z(a^z|\theta^z)\right)(1-d)i\left(a_p^z\middle|\theta^z\right)$. $E[\widetilde{B_p^z}|p \text{ has won}]$ refers to the ex post expected proposal given that the player has won the tender and is found as the conditional expectation: $E\left[\widetilde{B_p^z}\middle|\widetilde{B_p^z} < \widetilde{B_k^z}, \forall k \neq p\right] = \int_{-\infty}^{+\infty} \frac{x_p}{q_p^z} b_p^z(x_p) \prod_{k\in\mathcal{P}\setminus\{p\}}\left(1 - B_k^z(x_p)\right) dx_p$. $(1 + g_p(.))$ is the player-specific actual cost, and $i(.)$ the cost of investing. In order to arrive at the total expected pay-off for this state, the discounted player's instantaneous pay-offs of the future stages need to be added to the expected instantaneous pay-off.

3.4 Equilibrium identification

In order to find the Markov perfect equilibrium, one needs to identify the equilibrium action profile (i.e., each player's equilibrium investment and mark-up percentage) in each state. In each state, all players simultaneously optimize the total expected pay-off function, as stated in Section 3.2. A player's pay-off is dependent on the actions taken by the opponents, so we need to propose the conditions for the action equilibrium. The equilibrium for this stochastic game is derived by backward induction. Consequently, in each state, it is assumed that the players bid according to the equilibrium in subsequent stages. In the stochastic game with Z projects and defining the vector a_{-p} as the action profile of the opponents of player p, the first order conditions for optimality in the last project (i.e., project Z) are defined as:

$$\begin{cases} \dfrac{\delta\rho_p^Z(a_p^{Z*}|\theta^Z, a_{-p}^{Z*})}{\delta i(a_p^{Z*}|\theta^Z, a_{-p}^{Z*})} = 0 \\ \dfrac{\delta\rho_p^Z(a_p^{Z*}\,|\theta^Z, a_{-p}^{Z*})}{\delta m(a_p^{Z*}|\theta^Z, a_{-p}^{Z*})} = 0 \end{cases} \qquad \forall p \in \mathcal{P}, \forall \theta^Z \in \Theta^Z$$

With this information on the action equilibrium in stage Z, the action equilibrium of stage $z = Z - 1$ can be derived and as soon as $z = 0$, a strategy equilibrium for the sequential game has been derived. The first order conditions in all states θ^z with $z \in \{1, \dots, Z-1\}$ are:

$$\begin{cases} \frac{\delta \rho_p^z(a_p^{z*}|\theta^z, a_{-p}^{z*})}{\delta i(a_p^{z*}|\theta^z, a_{-p}^{z*})} + \Delta \sum_{k=1}^{P} \frac{\delta q_k^z(a_p^{z*}|\theta^z, a_{-p}^{z*})}{\delta i(a_p^{z*}|\theta^z, a_{-p}^{z*})} \pi_p^{z+1}(a^{z+1*}|\theta^{z+1}, k \text{ wins}) = 0 & \forall p \in \mathcal{P} \\ & \forall \theta^z \in \Theta^z \\ \frac{\delta \rho_p^z(a_p^{z*}|\theta^z, a_{-p}^{z*})}{\delta m(a_p^{z*}|\theta^z, a_{-p}^{z*})} + \Delta \sum_{k=1}^{P} \frac{\delta q_k^z(a_p^{z*}|\theta^z, a_{-p}^{z*})}{\delta m(a_p^{z*}|\theta^z, a_{-p}^{z*})} \pi_p^{z+1}(a^{z+1*}|\theta^{z+1}, k \text{ wins}) = 0 & z \in \mathcal{Z}\backslash\{Z\} \end{cases}$$

Since the derivative may not account for the border points of the investment and mark-up interval, one could also rely on the general definition for a Nash equilibrium in each state which states that none of the bidders has an incentive to deviate from his current action a_p^{z*}, given the action profile of the opponents a_{-p}^{z*}: $\forall p \in \mathcal{P}, \forall \theta^z \in \Theta^z, \forall z \in \mathcal{Z}, \forall a_p^z \in \mathcal{A}_p: \pi_p\left(a_p^{z*}\middle|\theta^z, a_{-p}^{z*}\right) \geq \pi_p(a_p^z|\theta^z, a_{-p}^{z*})$.

Finding this equilibrium is a hard problem and a heuristic approach is developed. The equilibrium of this dynamic programming problem is first determined for the states that are related to the final project in the pipeline. Based on a backward induction reasoning, the equilibria for each stage z are determined, assuming equilibrium play in the stages $\{z+1, \dots, Z\}$. So for a state variable $\theta^z = (e_1, e_2, \dots, e_P, Z - z)$, we want to derive an equilibrium action profile $a^* = (i_1, m_1, \dots, i_P, m_P)$, which is a vector with dimension $\mathbb{R}^{2*P}$. If no confusion is possible, subscripts and superscripts related to the state variable are omitted so as to avoid notational complexity. In order to determine the equilibrium, a straightforward best response heuristic is applied. In each iteration of the heuristic, the algorithm approaches the best response for each player sequentially, given the actions of the opponents. Once none of the bidders can improve their response given the opponents' actions, there is evidence that one might have arrived in an equilibrium. Academic literature only offers theoretical proofs for a limited number of circumstances for which the best response algorithm converges [6, 46]. Also in the context of the PPP procurement setting, we did not succeed in theoretically guaranteeing the convergence of the heuristic, while the computational results indicated convergence for the majority of the investigated cases. It is important to note that there is no guarantee that a pure strategy equilibrium exists in the case of an infinite number of actions. The algorithm looks into unique equilibria, but of course multiple equilibria might exist and the convergence towards a particular equilibrium is path-dependent. Therefore, the algorithm is executed for a predefined number of starting points. This repetitive structure accounts for the possibility of arriving in alternative equilibria. The pseudo-code for the algorithms can be found in Appendix 1.

3.4.1 Best response determination

Optimizing the pay-off function for a given set of actions for the opponents (indicated as a_{-p}) is a computationally intensive task because of the structure of the non-linear pay-off response function (Fig. 2). There are two decision variables for each player p that, together, represent the action a_p: the investment percentage i_p and the mark-up percentage m_p. The best response optimizes $\text{argmax}_{a_p} \pi_p^z\left(a_p \middle| \theta^z, a_{-p}\right) = \rho_p^z\left(a_p \middle| \theta^z, a_{-p}\right) + \Delta\mathcal{V}_p^{z+1}(a_p|\theta^z, a_{-p})$. In order to efficiently determine the global optimum, this paper uses an electromagnetism-like mechanism for global optimization as has been proposed by Birbil and Fang [4] and whose notation and procedure is utilized here. The attraction-repulsion mechanism of the heuristic succeeds in efficiently browsing through the entire search space and overcomes the danger of arriving in a local minimum, which could be the case when adopting the steepest ascent heuristic. The derivation of the best response is a two-step process. Given the action profile of the opponents a_{-p}, a set $\mathcal{T}$ of T solutions is initialized. A solution $t \in \mathcal{T}$ is represented as $x^t = (x_1^t, x_2^t)$, with $x_1^t \in [l_1, u_1]$ the coordinate that refers to the investment level and $x_2^t \in [l_2, u_2]$ the coordinate that refers to the mark-up level. l_1 and l_2 represent the lower bounds and u_1 and u_2 the upper bounds. For each initial point, the expected pay-off for player p, for whom the best response is derived, equals $f_p(x^t) = \pi_p(x^t|\theta^z, a_{-p})$. The best solution is stored as $f_p\left(x^{\text{best}}\right)$.

[FIGURE 2 TO BE INSERTED HERE]

3.4.1.1 Electromagnetism step

A charge is calculated for each point, based on the superposition principle of electromagnetism theory. The charge represents point i's power of attraction or repulsion and is calculated by: $q^t = \exp\left(-2 * \frac{\left(f_p(x^{\text{best}}) - f_p(x^t)\right)}{\sum_{k=1}^{T}\left(f_p(x^{\text{best}}) - f_p(x^k)\right)}\right)$. In the next step, the total forces on each point are computed. For the pairwise force calculations, the point that has a higher expected pay-off, attracts the other point, while the point with a worse pay-off repels the other point. The forces and moves are calculated for all but the currently best found point, in order to keep the information of the current best point. Furthermore, let F_n^{tu} represent the force exerted by point u on point t for coordinate n, with $n = 1$ for the investment direction and $n = 2$ for the mark-up direction. Finally, F_n^t refers to the total force exerted by the other points for coordinate n. As a result, we obtain $F^t = \{F_1^t, F_2^t\} = \{\sum_{u \in \mathcal{T} \setminus \{t\}} F_1^{tu}, \sum_{u \in \mathcal{T} \setminus \{t\}} F_2^{tu}\}$ and the force is represented as:

$$F^t = \sum\nolimits_{u \in \mathcal{T} \setminus \{t\}} \frac{(-1)^{w^{tu}} (x^u - x^t) q^t q^u}{\|x^u - x^t\|^2}$$

with $w^{tu} = 0$ if $f_p(x^t) < f_p(x^u)$ and $w^{tu} = 1$ if $f_p(x^t) \geq f_p(x^u)$. In order to move the points, the force vector F^t is normalized into $\bar{\bar{F}}^t = F^t / \|F^t\|$ and a random step length η is selected from $U(0,1)$ so that there is a non-zero probability to move to the unvisited regions along the direction of F^t. The normalization of the force vector ensures that the new solution is located in the feasible region. In conclusion, all but the previously found best point are moved so that for each $t \in \mathcal{T} \setminus \{\text{best}\}$ and $n \in \{1,2\}$, the respective coordinate becomes:

$$x_n'^t = \begin{cases} x_n^t + \eta \bar{\bar{F}}_n^t (u_n - x_n^t) & \text{if } \bar{\bar{F}}_n^t > 0 \\ x_n^t - \eta \bar{\bar{F}}_n^t (x_n^t - l_n) & \text{if } \bar{\bar{F}}_n^t \leq 0 \end{cases}$$

The procedure is iterated a predefined number of times with the newly found coordinates and subsequently, the local search algorithm is executed.

3.4.1.2 Example

Consider a two-player situation for which we want to determine the best response for $p = 1$ and given the action (1,20) for player 2 which refers to a 1% investment percentage and a 20% mark-up percentage. Table 1 reports the algorithm values for $T = 4$. The pay-off of player 1 for each point t is given by $f_1(x^t)$, so that the best pay-off is obtained in point 1. The sum of all the differences between the points' pay-off values and the best pay-off is 0.035589 and the charge for point $t = 2$ is found as $\exp\left(-2 * \frac{0.046640 - 0.040448}{0.035589}\right)$. The distance between point 2 and 3 is 4.4778 and the force that point 3 exerts on point 2 equals $F_1^{23} = (1.7 - 3.5) * \frac{0.7484 * 0.7061}{4.4778} = -0.2125$ for the investment coordinate and $F_2^{23} = (16.1 - 20.2) * \frac{0.7484 * 0.7061}{4.4778} = -0.4839$ for the mark-up coordinate. The forces on point 2's investment coordinate from the points 1 and 4 are -0.1022 and 0.0419 respectively, so that the total force $F_1^2 = -0.2728$. After normalizing and generating η=0.4 as the random variable and defining $(l_1, l_2) = (0,0)$ and $(u_1, u_2) = (5,30)$, the new investment coordinate for $t = 2$ equals $3.5 + 0.4 * (-0.5725) * 3.5 = 2.6985$. The electromagnetism procedure is iterated with the previously best found point ($t = 1$) and with the three newly found points.

[TABLE 1 TO BE INSERTED HERE]

3.4.1.3 Local search step

The points that result from the electromagnetism step are the input for a local search procedure around each of these points. In each iteration, the investment and mark-up coordinate are modified by applying a random transformation around the original point using a step length vector α and a randomly generated point κ from a uniform distribution that determines the amount and direction of the perturbation. If the new point results in a higher pay-off, the coordinates are updated and the local search procedure continues from the newly found point. In the end, the best known solution is updated.

3.4.1.4 Example

Let us start from the initial point $(1.48{,}21.37)$ while the action for the second player is $(1{,}20)$. This results in a pay-off of 0.049456. Assume that the step length for the investment is 0.3 and 2 for the mark-up. For each coordinate, a random variable is generated from $U(0{,}1)$: $\kappa = 0.09$ for the investment coordinate and $\kappa = 0.86$ for the mark-up coordinate. In order to allow for positive and negative perturbations, we subtract 0.5 from the randomly generated number, so the new investment coordinate becomes $1.48 + (0.09 - 0.5) * 0.3 = 1.36$ and for the mark-up shift, the mark-up in the new point equals $21.37 + (0.86 - 0.5) * 2 = 22.09$. The point $(1.36{,}22.09)$ leads to a pay-off of 0.0501 and consequently the local search continues from this outperforming point.

3.4.2 Equilibrium selection

For a starting point that is randomly selected from the entire action profile space, the best response heuristic is executed until the convergence criterion is satisfied. Convergence occurs as soon as no significant pay-off improvements have been registered in a pre-defined number of loops and as long as a minimum number of loops $BRMINREP$ has been executed. Moreover, the algorithm stops when the number of loops reaches $BRMAXREP$. The convergence is path-dependent, so the algorithm is repeated for R starting points. Afterwards, the R resulting action profiles are clustered according to a distance criterion resulting in C clusters. Points are sequentially assigned to the cluster for which the distance between the point and the average coordinates of the cluster is minimal and smaller than $MAXDIST$. Alternatively, a new cluster is formed. Within each cluster, the coordinate-by-coordinate average of all points belonging to the cluster are calculated together with the pay-off profile. Clusters with a number of points smaller than $MINSUPPORT$ are removed as these might have resulted from a local optimum. The second step in the equilibrium selection process compares the averages of the clusters and removes any clusters that are dominated by other clusters (i.e., a cluster that has higher pay-offs for all players). Subsequently, the clusters are ranked according to the sum of the average pay-offs of all players and the cluster with the highest total pay-off is moving to the refinement stage. The refinement stage executes the best response heuristic again to confirm whether the particular action profile is an equilibrium. If this is confirmed, the resulting reported equilibrium for the state is stored and is used to compute the equilibria in the stage $z - 1$.

4. Experimental results

4.1 Experimental setting

The computer experiments have been executed in Microsoft Visual Studio 2010 and Wolfram Mathematica 8.0 performs the expected pay-off calculations. As the risk profile of PPP projects deserves major interest, we are mainly interested in the effect of the level of uncertainty, which is reflected by the knowledge impact parameters γ_i (i.e., variability due to a lack of investment) and γ_e (i.e., variability due to a lack of experience). Moreover, it is necessary to account for the computational effort that the equilibrium determination requires. Therefore, three sets of scenarios have been developed. Each scenario is executed for every unique combination of experience levels. Sections 4.3 and 4.4 give insights into the general dynamics of the equilibrium and rely on the four scenarios that are determined by the parameter values of Table 2. We will refer to these as follows: Sc.1 ($\gamma_i = \gamma_e = 0.1$), Sc. 2 ($\gamma_i = 0.1, \gamma_e = 0.05$), Sc. 3 ($\gamma_i = 0.05, \gamma_e = 0.1$) and Sc. 4 ($\gamma_i = \gamma_e = 0.05$). Secondly, Section 4.5 covers a case (Sc. 5) with extreme project risks ($\gamma_i = 0.2$, $\gamma_e = 0.1$). Thirdly, the impact of introducing government compensation in Sc. 1 and Sc. 5 is assessed in Section 4.6. Finally, Section 4.7 highlights some noteworthy robustness aspects from a more extensive sensitivity analysis of the cost impact related parameters in a setting with only two experience intervals ($e_u = 2$). Besides, after trading off computation times and the level of accuracy and assessing the speed of convergence, the algorithm variables are tuned as shown in Table A1.1 of Appendix 1.

[TABLE 2 TO BE INSERTED HERE]

4.2 Algorithm performance

For each state of the game, the algorithm looks for the state-specific action equilibrium. A scale with $e_u = 5$ intervals leads to six experience levels, which results in 21 experience vectors (0,0), (0,1), (0,2), (0,3), (0,4), (0,5), (1,1), (1,2), …, (5,5) in a two-player setting. Equivalently, a three-player setting with five intervals results in 56 experience vectors. Convergence occurs when the algorithm stops before the maximum number of replications has been reached. For Sc. 1-4, the convergence criterion is met up to a three-project pipeline ($Z = 3$) in the three-player setting. In the two-player setting, convergence issues appeared solely in a three-project pipeline for subgames (0,4) and (0,5) in which experienced players are randomizing within a small area of the action profile space of Sc. 1 between actions without investment on the one hand and with a small investment (0.5%) and a slightly higher (1%) mark-up.

In the convergent cases, the search procedure converges already after less than $BRMINREP$ loops towards a specific search region in the action profile matrix. Changing the players' sequence of best response selection does not modify this tendency. In the first loops of the best response algorithm, the electromagnetism is mainly attributable for the selection of the best response, while in later iterations the local search procedure will be mainly responsible for the determination of the optimizing step. In the vast majority of the cases, the algorithm reports a single cluster. Only in the two- and three-project context of Sc. 1 and Sc. 2, two equilibria are apparent for the mature markets (4,4,5) and (4,5,5).

4.3 Impact of the pipeline on the procurement of the first project

[TABLE 3 TO BE INSERTED HERE]

Table 3 reports the statistical results of the scenario-by-scenario analysis for the strategic actions in the tender for the first project in a single-, a two- and a three-project environment with two and three players. Within the stochastic game notation, the table compares the actions of the states for a constant experience vector e, but for a variable number of remaining projects in the pipeline. So, if a state is represented as $\theta^z = (e_1, e_2, Z - z)$, this section compares the actions for $\theta^1 = (.,.,0)$ with $\theta^1 = (.,.,1)$ and $\theta^1 = (.,.,2)$. A parametric paired t-test has been used to study the paired observations. In order to guarantee the results' robustness, the output of the non-parametric Wilcoxon signed rank test is also reported. The statistical tests confirm the initial expectation that mark-ups are lowered when extra projects are introduced both in the two-player as well as in the three-player case. The drop in mark-ups is the greatest for the inexperienced players, resulting in more competition within the heterogeneous market. An analysis of the investment dynamics tends to point towards decreasing investment percentages when more projects are in the pipeline. Conditioning on the experience levels for the comparison of a three- versus a single-project case with three players indicates an insignificant average absolute drop of

0.001% for inexperienced players (p-value=0.106), towards a significant average 0.02% drop (p-value=0.004) for players with $e_1 = 1$ and $e_1 = 2$ and a drop of 0.045% for players with maximum experience (p-value=0.0003). Looking at the two-player environment in which the investments are generally higher, the results show a significant decrease of 0.24% and 0.05% for players with experience levels 0 and 1 respectively (p-values 0.001 and 0.025), but do report a significant increase for players from experience level 3 onwards.

Eventually, the decreasing mark-ups also lead to lower bidder's pay-offs for the first project and a lower project procurement cost for the government. According to the aggregated scenario outcomes, the mark-up and consequently also the pay-off drop is largest for the inexperienced players and in the subgames where mainly inexperienced players are involved. On average, a decrease of 13.53% in the pay-off has been reported in the two-player case and 19.35% in the three-player case when moving from a single-project to a three-project pipeline. Additionally, inexperienced players are sometimes willing to suffer a loss in the first project in order to win the project and obtain a greater experience level for the next project. In general, the percentage-wise drop in pay-offs has a negative trend in the experience level. The reduced mark-ups have positive repercussions on the government expenditures. The government cost for the first project in a three-project pipeline is 1.12% lower in two-player subgames and 0.92% lower in three-player subgames than in the case without a pipeline. In immature markets, savings of 5.14% and 4.13% might be realized in two- and three-player settings respectively.

Furthermore, an ANOVA study of the investment percentage in function of the model parameters and the state-defining values with first-order interaction effects could not support significance at the 5% level of any of the terms related to the number of projects in the pipeline in the three-player case. In the two-player model, both the interaction term of the experience level e_1 and the number of projects in the pipeline as well as the interaction term of the competitor's experience level e_2 and the pipeline length are highly significant (p-values equal $4.6*10^{-7}$ and $9.8*10^{-5}$ respectively). The ANOVA models for the mark-ups, on the other hand, reveal the main effect and the interaction effects of the pipeline variable. We refer to Appendix 2 for the full ANOVA results. The ANOVA tests also underline the interaction with the players' experience vector that defines the state variable. Tables 4 and 5 show the action dynamics in the bidding behavior for the first project for a player with experience level e_1 with respect to a variable number of projects in the pipeline. Only the scenarios that involve players with experience level zero up to two are reported, but analogous results are apparent for the remaining scenarios. Moving towards a multi-project environment considerably reduces the mark-ups for the inexperienced players. According to these tables, only players with a competitive advantage over all their competitors (e.g., the subgames (e_1, e_2, e_3)=(1,0,0), (2,0,0) or (2,0,1) and (e_1, e_2)=(1,0), (2,0) or (2,1)) have a tendency to invest more in a multi-project environment than in a single-project setting, which means that players who were already in a beneficial position opt to strengthen their advantage even more. The cases in which a player has an experience level that is equal to or lower than at least one of his opponents point towards decreasing investment percentages.

[TABLES 4 AND 5 TO BE INSERTED HERE]

4.4 Impact of a pipeline on the average expected bidding behavior

Section 4.3 solely looked at the bidding dynamics of the first project of the pipeline. A second approach to analyze the dynamics of the bidding behavior compares the average expected investment and mark-up percentages of the players over the entire pipeline. Given the optimal action profiles a^{*z}, the average investment level over the entire pipeline is calculated by taking the sum of the investment levels of each state $\theta^z = (.,.,Z-z)$ weighted with the probability that this state occurs. Consider the three-project pipeline and initial experience vector $(e_1, e_2) = (0{,}0)$ of Fig. 1. The expected average investment percentage for player 1 equals $(0.6\% + 0.43(1.3\% + 0.50 * 1.6\% + 0.50 * 1.3\%) + 0.57(0.6\% + 0.43 * 1.3\% + 0.57 * 0.8\%))/3 \approx 0.9\%$. These averages are then compared to the situation in which a single project is tendered three times, which means that only the actions and transition probabilities of the state variables $\theta^z = (.,.,1)$ are considered. The average expected pay-offs and government cost may be obtained in a similar fashion. Table 6 reports the statistical analysis of the differences and agrees with the previously stated findings. The former mark-up results are confirmed so that, from a procurement perspective, bidding becomes more aggressive in the case of a project pipeline than when projects are tendered without communicating the pipeline. The analysis of the average investment efforts in the experiment show that the differences in the dynamics are attributed to the competitive position of the player with respect to the opponents. In the model, pre-tender investment results in a reduction of the uncertainty, so that sometimes more accurate estimations point towards higher project costs. Consequently, bidders might prefer to be less informed and play with the mark-up, win a project and use the gained experience for future projects.

[TABLE 6 TO BE INSERTED HERE]

4.5 High-risk situation

A change of the γ_i parameter of Sc. 1 to a value of 0.2 refers to a case with high project-related risk (Sc. 5). The observation of the three-player equilibrium results has led to three findings. Firstly, not all subgames could guarantee convergence. In the subgames (0,0,4) and (0,0,5) with a single project for instance, the inexperienced players are looping over strategy profiles in which they both participate or one stays out. Consequently, we might expect that there is an equilibrium in mixed strategies. As the results of a multi-project pipeline rely on the outcome of the single-project case, this non-convergence effect acts as a bullwhip to earlier states of the stochastic game. Secondly, among all scenarios, there are only two cases for which the equilibrium allows participation for all players for the single-project case: subgame (0,0,0) with a 2.1% investment and a 25% mark-up for all participants and subgame (1,1,1) with a 2.4% investment and a 21% mark-up. In all other subgames, the reported equilibrium always suggests that one player should stay out of this engagement (i.e., 0% investment and 50% mark-up). Thirdly, adding extra projects has the same consequences as described earlier, but players who applied a no-participation action in the single-project case, will still prefer to stay out of the market. As a result, the model argues that it might be unsustainable to invite three contractors for the bid preparation stage in a high-risk project environment. Therefore, the equilibria of a two-player setting are of very high interest. In this vein, the previous results seem to be robust: everyone participates, mark-ups decrease in the length of the pipeline and players with a competitive advantage over their opponent tend to invest more, but the investment gaps are small.

4.6 Government reimbursement

In general, one might say that the introduction of a pipeline leads to fiercer competition from a mark-up perspective. Nevertheless, the inexperienced bidders are still significantly less willing to invest in research, which is especially the case in a three-player setting. Therefore, a government reimbursement might help to level the playing field from an investment perspective. In this way the government gives all bidders an opportunity to enter the market which would prevent oligopolistic behavior or hidden collusion among mature bidders for instance. The government determines a compensation percentage d that reflects the fraction of the investment efforts that is reimbursed to all losing bidders. Table 7 summarizes the main characteristics of the average bidding equilibrium results of a three-project pipeline for Sc. 1 and high-risk Sc. 5 in a two-player setting and for Sc.1 in the three-player case.

[TABLE 7 TO BE INSERTED HERE]

If two players are prequalified, the investment willingness is relatively symmetric for both players, both in the low-risk as well as in the high-risk case. An introduction of a compensation raises the investment willingness. The average mark-up percentages tend to decrease to the level of 40% compensation in the low-risk case, while they increase from 40% compensation onwards in the high-risk setting. Of course, this reimbursement involves additional expenditures for the government, but these extra costs can be partly offset by the savings that result from the pipeline concept. Moreover, both players have a considerable investment willingness, making the introduction of a large reimbursement policy obsolete, because it would only increase the government cost and inflate the contractors' profits.

Three-player competition will generally lead to fiercer competition and a lower government cost. Nevertheless, Table 7 shows that without compensation, it rarely happens that all players of a particular subgame invest. The use of compensation overcomes this dynamic in the majority of the subgames, resulting in a higher average investment. In the subgames where inexperienced players are competing against two mature contractors though, the inexperienced player moves to an equilibrium action in which he does not participate. On average, a compensation of 40% together with a pipeline of three projects is cheaper than a situation without compensation and without a pipeline. So, in the case governments aim for three competing consortia, a combination of both policies leads to promising results. Section 4.5 highlights the mixed equilibrium behavior of the high-risk case, which prevents us from giving a full analysis of the compensation mechanism. Nevertheless, the experiments show that an 80% reimbursement levies the convergence issue and would ensure that all players of the subgame invest in 46 out of the 56 subgames. For the ten remaining vectors, the 80% reimbursement does not serve as a sufficient incentive to make the inexperienced player who is facing two players with an e_p of at least 2 to invest more. Nevertheless, in this high-risk setting, attributing these compensations requires additional government expenditures that may only be partly offset by the reduced mark-ups in case of a pipeline, so governments might prefer to only prequalify two contractors in a high-risk set-up. Finally, since the investment percentages reflect the willingness of the contractor to invest, it should evidently be the government's priority to reduce the (non-value adding) investment requirements.

4.7 Additional scenarios

In order to investigate the robustness of the results, a full factorial 2^4 design has been set up with two levels (i.e., 0.05 and 0.1) for the parameters γ_i, γ_e, β_i and β_e. The scenarios have been executed for each subgame (i.e., every possible combination of the experience levels) in a two- and three-player setting with $Z = \{1,2,3\}$. For this robustness study, only two experience intervals are considered, so that $e_u = 2$ and a player's experience level is 0, 1 or 2.

First of all, the results for the states with experience vectors (0,2) and (0,0,2) often did not converge (i.e., in 11 out of 16 scenarios and 5 out of 16 scenarios respectively). In these instances, the algorithm loops between different action profiles. The inexperienced players mix their choice between no investment and a high mark-up and a moderate investment with a low mark-up. The experienced player responds accordingly with a low or a high mark-up. It is especially when the parameters that are related to the experiential advantage (γ_e and β_e) are high that these convergence issues occur. Nevertheless, the dynamics of the previous sections are confirmed by this sensitivity study. Mark-ups are decreasing in the number of projects, regardless of the number of players and their respective experience levels. Concerning the investments, the players who have a competitive advantage in the initial stage of the game increase the investment willingness when the project pipeline grows larger.

The limited number of experience intervals underlines the importance of winning a project early in the pipeline. Therefore, the percentage-wise impact of the pipeline on the mark-ups is greater than when $e_u = 5$. According to the aggregated scenario outcomes, the pay-off drop for the first project of a three-project pipeline is largest for the inexperienced players and in subgames in which inexperienced players are involved. On average, a decrease of 73% in the pay-off has been reported in the two-player case and 109% in the three-player case when moving from a single-project to a three-project pipeline. Consequently, inexperienced players are willing to suffer a loss in the first project in order to win it and obtain a greater experience level for the next project. The pay-off drops for the medium experienced player equal 20% and 33% for the two- and three-player setting respectively and for the player with maximum experience, this amounts to an 8% and a 17% drop. As a result, the combined sensitivity results also confirm the reduction in the government expenditures. A three-project pipeline reduces the procurement cost of the first project with 1.7% in subgames with two players and with 2.2% in three-player subgames.

4.8 Practitioners' view

Along the course of the research project, a number of international contracting companies, Belgian and Australian public institutions and a set of advisory firms have been interviewed to provide feedback on the models and the results. They all believe in the positive impact of a project pipeline and its capability of increasing competition. For contractors, it is a means to spread budgets and risk over multiple projects, although mature companies believe that the pipelines will limit their profits because of the stronger competitive forces. Another consequence of the theoretical findings is that governments might consider to split up large-scale infrastructure projects into a sequence of smaller projects. As a result, also small contractors, for whom the bid costs might become a financial burden, benefit from entering the market. If a government wants to ensure to keep the competition going and to avoid a situation in which contractors become too comfortable in their mature position, they might have to put effort in cementing the pipeline so that contractors have sufficient belief in the trustworthy pipeline. In addition, practitioners also underline the importance of reducing the bidding cost in general and of investing in the efficiency of the procurement process [25].

5. Conclusion

This paper studies the impact of a pipeline on the dynamic bidding behavior in a PPP setting. Governments do not always have the opportunity to foresee the budget for these risky and expensive projects in the long term. Nevertheless, public institutions in Canada and Australia for instance do succeed in maintaining and cementing a considerable project pipeline. The dynamic procurement format has been translated into a sequential procurement auction model in which contractors that are heterogeneous in their cost probability distributions will determine their sequential strategy over the course of a commonly known pipeline and a constant set of bidders. These bidders can, at each stage, modify their ex ante expected cost distributions by their investment decision. Being granted a project results in a knowledge and cost advantage for future projects. An experimental study has been performed to gain initial insight into the optimal strategy. A method to gradually derive the Markov equilibrium has been presented. Under the assumptions that outline the scope of the paper, the results support the hypothesis that mark-ups tend to decrease when more projects are included in the pipeline, regardless of the number of bidders. In this vein, the mark-up result is in line with the contemporary sequential auction theory literature. Consequently, bidders are willing to accept lower profits if this might lead to future benefits. As a consequence, the government procurement cost tends to have a decreasing nature in the number of projects. Looking at the upfront investment willingness, there has been only limited support for an increasing trend for players who dispose of a competitive advantage at the start of the pipeline. In all other cases, the investment percentages tend to decrease. Therefore, an investment reimbursement might still be necessary to trigger the consortia's enthusiasm to perform more upfront research. This levels the competition and reduces the risk of a contractor's default, which would always come at the government's expense due to the societal value of PPP projects. The extra cost of investment reimbursements might be partly offset by the reduced mark-ups that result from the pipeline. Nevertheless, in cases with considerable project-specific risks, it might be better to only prequalify two contractors to engage in the expensive tendering process. Contractors, public institutions and advisors that have provided comments on these results all believe in the importance of a project pipeline to increase competition and to attract new or overseas consortia that are willing to invest more money and resources to enter a market with a long-term perspective.

Methodology-wise, this study has some limitations that require further consideration. Firstly, the findings are theoretical, are based on stringent assumptions and essentially need to be validated in a practical environment. As a PPP setting is highly competitive, bidding data are scarce to fully validate the model. Furthermore, a relaxation of the assumptions that pre-tender investment does not contribute to future knowledge and that contractors have unlimited capacity offers an opportunity for further research. Thirdly, for some scenarios, the heuristic did not converge in all states of the game. As earlier stages of the pipeline rely on the optimal bidding behavior of later stages, this limits the study of the entire scenario. Subsequently, the flexibility of the model trades off against the manageability of the analytical calculations. Nevertheless, this shortcoming together with the highly relevant and state-of-the-art bidding framework, leaves opportunities for research in related fields like mathematical auction theory and algorithmic game theory. Last but not least, the magnitude and the financial burden of PPP projects often inhibit to bid on more than one or two projects. Further research could focus on the interdependency of capacity constraints and the resulting equilibria.

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References

[1] Ahadzi M, Bowles G (2004) Public-private partnerships and contract negotiations: An empirical study. *Construction Management and Economics* 22 967-978.

[2] Aziz AMA (2007) Successful delivery of public-private partnerships for infrastructure development. *Journal of Construction Engineering and Management* 133 918-931.

[3] Bergemann D, Välimäki J (2002) Information acquisition and efficient mechanism design. *Econometrica* 70 1007-1033.

[4] Birbil SI, Fang SC (2003) An electromagnetism-like mechanism for global optimization. *Journal of Global Optimization* 25 263-282.

[5] Branco F (1997) Sequential auctions with synergies: An example. *Economics Letters* 54 159-163.

[6] Brun O, Prabhu BJ, Seregina T (2013) On the convergence of the best-response algorithm in routing games. In *Proceedings of the Seventh International Conference on Performance Evaluation Methodologies and Tools*, Turin (Italy), 136-144.

[7] Carrillo P, Robinson H, Foale P, Anumba C, Bouchlaghem D (2008) Participation, barriers and opportunities in PFI: The United Kingdom experience. *Journal of Management in Engineering* 24 138-145.

[8] Chan APC, Lam PTI, Chan DMW, Cheung E, Ke Y (2010) Critical success factors for PPPs in infrastructure developments: Chinese perspective. *Journal of Construction Engineering and Management* 136 484-494.

[9] De Clerck D, Demeulemeester E (2014) *Towards a more competitive PPP procurement market: A game-theoretical analysis.* FEB Research report KBI_1408, 59pp. Leuven (Belgium): KU Leuven – Faculty of Economics and Business.

[10] De Silva DG, Jeitschko TD, Kosmopoulou G (2005) Stochastic synergies in sequential auctions. *International Journal of Industrial Organization* 23 183-201.

[11] Elmaghraby W (2003) The importance of ordering sequential auctions. *Management Science* 49 673-682.

[12] Engelbrecht-Wiggans R (1994) Sequential auctions of stochastically equivalent objects. *Economics Letters* 44 87-90.

[13] Farias V, Saure D, Weintraub GY (2012) An approximate dynamic programming approach to solving dynamic oligopoly models. *RAND Journal of Economics* 43 253-282.

[14] Harstad RM (2010) Auctioning the right to choose when competition persists. *Decision Analysis* 7 78-85.

[15] Ho SP (2005) Bid compensation decision model for projects with costly bid preparation. *Journal of Construction Engineering and Management* 131 151-159.

[16] Hörner J, Jamison J (2008) Sequential common-value auctions with asymmetrically informed bidders. *Review of Economic Studies* 75 475-498.

[17] Jefferies M (2006) Critical success factors of public private sector partnerships: A case study of the Sydney SuperDome. *Engineering, Construction and Architectural Management* 13 451-462.

[18] Jofre-Bonet M, Pesendorfer M (2003) Estimation of a dynamic auction game. *Econometrica* 71 1443-1489.

[19] Jofre-Bonet M, Pesendorfer M (2014) Optimal sequential auctions. *International Journal of Industrial Organization* 33 61-71.

[20] Kannan KN (2010) Declining prices in sequential auctions with complete revelation of bids. *Economics Letters* 108 49-51.

[21] Katehakis MN, Puranam KS (2012) On bidding for a fixed number of items in a sequence of auctions. *European Journal of Operational Research* 222 76-84.

[22] Katzman B (1999) A two stage sequential auction with multi-unit demands. *Journal of Economic Theory* 86 77-99.

[23] Ke Y, Wang S, Chan APC, Cheung E (2009) Research trend of public-private partnership in construction journals. *Journal of Construction Engineering and Management* 136 138-150.

[24] King M, Mercer A (1990) The optimum markup when bidding with uncertain costs. *European Journal of Operational Research* 47 348-363.

[25] KPMG (2010) *PPP procurement: Review of barriers to competition and efficiency in the procurement of PPP projects.* KPMG Corporate Finance (aust) Pty Ltd, 65pp.

[26] Maskin E, Tirole J (2001) Markov perfect equilibrium, I: Observable actions. *Journal of Economic Theory* 100 191-219.

[27] Menezes FM, Monteiro PK (2004) Auctions with synergies and asymmetric buyers. *Economics Letters* 85 287-294.

[28] Milgrom P, Roberts J (1990) Rationalizability, learning, and equilibrium in games with strategic complementarities. *Econometrica* 58 1255-1277.

[29] Milgrom P, Weber R (1982) A theory of auctions and competitive bidding. *Econometrica* 50 1089-1122.

[30] Monderer D, Shapley LS (1996) Potential Games. *Games and Economic Behavior* 14 124-143.

[31] Naert PA, Weverbergh M (1978) Cost uncertainty in competitive bidding models. *Journal of the Operational Research Society* 29 361-372.

[32] Nisan N, Roughgarden T, Tardos E, Vazirani VV (2007) *Algorithmic Game Theory*. Cambridge University Press, New York (USA), 754pp.

[33] Oren SS, Rothkopf MH (1975) Optimal bidding in sequential auctions. *Operations Research* 23 1080-1090.

[34] Persico N (2000) Information acquisition in auctions. *Econometrica* 68 135-148.

[35] Pitchik C (2009) Budget-constrained sequential auctions with incomplete information. *Games and Economic Behavior* 66 928-949.

[36] Reiβ JP, Schöndube JR (2010) First-price equilibrium and revenue equivalence in a sequential procurement auction model. *Journal of Economic Theory* 43 99-141.

[37] Said M (2011) Sequential auctions with randomly arriving buyers. *Games and Economic Behavior* 73 236-243.

[38] Shapley LS (1953) Stochastic games. In *Proceedings of the National Academy of Sciences of the United States of America* 39 1095-1100.

[39] Shi X (2012) Optimal auctions with information acquisition. *Games and Economic Behavior* 74 666-686.

[40] Sørensen ST (2006) Sequential auctions for stochastically equivalent complementary objects. *Economics Letters* 91 337-342.

[41] Sureka A, Wurman PR (2005) Using tabu best-response search to find pure strategy Nash equilibria in normal form games. In *Fourth International Joint Conference on Autonomous Agents and Multiagent Systems*, Utrecht (NL), 1023-1029.

[42] Takano Y, Ishii N, Muraki M (2014) A sequential competitive bidding strategy considering inaccurate cost estimates. *Omega* 42 132-140.

[43] Tang L, Shen Q, Cheng EWL (2010) A review of studies on public-private partnership projects in the construction industry. *International Journal of Project Management* 28 683-694.

[44] Tserng HP, Russell JS, Hsu CW, Lin C (2012) Analyzing the role of national PPP unit in promoting PPP – Using new institutional economics and case study. *Journal of Construction Engineering and Management* 138 242-249.

[45] Von der Fehr NHM (1994) Predatory bidding in sequential bidding. *Oxford Economic Papers* 46 345-356.

[46] Vorobeychik Y, Wellman MP (2008) Stochastic search methods for Nash equilibrium approximation in simulation-based games. In *Proceedings of Seventh International Conference on Autonomous Agents and Multiagent Systems,* Estoril (Portugal), 1055-1062.

[47] Weber RJ (1983) Multiple-object auctions. In R. Engelbrecht-Wiggans, M. Shubik, R.M. Sterk, eds., *Auctions, bidding, and contracting: Uses and theory* (New York University Press) 165-191.

[48] Wolfram CD (1998) Strategic bidding in a multi-unit auction: An empirical analysis of bids to supply electricity in England and Wales. *RAND Journal of Economics* 29 703-725.

[49] Yildirim H (2004) Piecewise procurement of a large-scale project. *International Journal of Industrial Organization* 22 1349-1375.

[50] Yuan J, Wang C, Skibniewski MJ, Li Q (2012) Developing key performance indicators for public-private partnership projects: Questionnaire survey and analysis. *Journal of Management in Engineering* 28 252-264.

[51] Zeithammer R (2007) Research note – Strategic bid-shading and sequential auctioning with learning from past prices. *Management Science* 53 1510-1519.

[52] Zeithammer R (2009) Commitment in sequential auctioning: Advance listings and threshold prices. *Journal of Economic Theory* 38 187-216.

Captions of tables and figures

Table 1 Illustration of the electromagnetism heuristic

Table 2 Parameter values for the computer experiment

Table 3 Average comparisons and associated p-values of the first stage of a two- or three-stage environment with respect to a single-stage environment

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Table 5 Actions for the first project in a pipeline with Z stages for player with initial experience level e_1 and competition given by e_2. Only the scenarios that involve players with $e_p \leq 2$ are reported.

Table 6 Scenario-by-scenario comparison of the average strategic behavior in the case of a three-project pipeline and the case with three times tendering a single project

Table 7 Aggregate results of the impact of government reimbursement on the equilibrium outcome of a three-project pipeline
[a] Number of subgames in which at least one player does not participate. The total number of subgames is 21 for the two-player setting and 56 for the three-player setting
[b] Number of subgames in which all players have investment levels greater than 0%
[c] Average cost of tendering three times a single project consecutively
[d] An inexperienced player does not invest for the first project if he is playing against e_2= 4 or 5 and Z = 3
[e] Player with experience level 0 in subgames (0,4,4), (0,4,5) and (0,5,5) does not participate when d=40%. Moving to 60% and 80% also adds (0,3,5) and (0,3,4), respectively, to the set of no-participation subgames.

Table A1.1 Algorithm parameters used in the experiment
Table A2.1 ANOVA output for the actions in the first project of a Z-project pipeline for a player with experience e_1 and the opponents' experience vector (e_2, e_3) as a factor variable

Table A2.2 ANOVA output for the actions in the first project of a Z-project pipeline for a player with experience e_1 and the opponent's experience level e_2 as a factor variable

Fig. 1 Example of the sequential strategy for two players and three projects with $e_u = 5$ and the parameters according to Sc. 1

Fig. 2 Example of the response function $\pi_1^1\left(a_1 \middle| \theta^1 = (1,1,2,0), a_{-1} = (1\%, 20\%, 1\%, 20\%)\right)$

Appendix 1: Equilibrium derivation algorithm

Algorithm 1: MainAlgorithm()
1: Generate R random action profiles $a^r \in \mathcal{A}$ for $r \in \{1,2,\dots,R\}$
2: for $r = 1$ to R do
3: BestResponseAlgoritm(a^r)
4: end for
5: EvaluateEquilibria()

Algorithm 2: BestResponseAlgorithm(a)
$BRMAXREP$= maximum number of loops
$BRMINREP$= minimum number of loops
$conv$=convergence threshold

$Counter \leftarrow 0$
for $k_1 = 1$ to $BRMAXREP$ do
$o \leftarrow a$
for $p = 1$ to P do
$a_{-p} \leftarrow a \ \setminus\{a_p\}$
SelectBestResponse(p, a_{-p})
$a_p \leftarrow x^{\text{best}}$
end for
If $\|o - a\|^2 < 0.00001$ do $counter \leftarrow counter + 1$
If ($k_1 > BRMINREP$ & $counter > conv$): break
end for

Algorithm 3: SelectBestResponse(p, a_{-p})
$EMREP$: number of electromagnetic iterations
$LSREP$: number of local search iterations
$\alpha = (\alpha_1, \alpha_2)$: step length for local search procedure

Generate T random actions $x^t \in \mathcal{A}_p$ for $t \in \{1,2,\dots,T\}$ and calculate $f_p(x^t) \leftarrow \pi_p\left(x^t \middle| \theta^z, a_{-p}\right)$
For $k_2 = 1$ to $EMREP$ do
$x^{\text{best}} \leftarrow \text{argmax}\{f_p(x^t), \forall x^t \in \mathcal{T}\}$
For all $x^t \in \mathcal{T}$ do
Calculate charges q^t
$F^t \leftarrow 0$
End for
For all $x^t \in \mathcal{T}\setminus\{x^{\text{best}}\}$ do
For all $x^u \in \mathcal{T}$ do
If $f_p(x^t) < f(x^u)$ do $F^t \leftarrow F^t + (x^u - x^t)q^t q^u/\|x^u - x^t\|^2$
Else $F^t \leftarrow F^t - (x^u - x^t)q^t q^u/\|x^u - x^t\|^2$
End for
End for
For all $x^t \in \mathcal{T}\setminus\{x^{\text{best}}\}$ do
$\eta \leftarrow U(0,1)$
$F^t \leftarrow F^t/\|F^t\|$
For $n = 1$ to 2 do
If $F_n^t > 0$ do $x_n^t \leftarrow x_n^t + \eta F_n^t(u_n - x_n^t)$
Else $x_n^t \leftarrow x_n^t - \eta F_n^t(x_n^t - l_n)$
End for
End for
End for
For all $x^t \in \mathcal{T}$ do
For $k_3 = 1$ to $LSREP$ do
$y \leftarrow x^t$
For $n = 1$ to 2 do
$\kappa \leftarrow U(0,1)$

$y_n \leftarrow y_n + \alpha_n(\kappa - 0.5)$
End for
If $f_p(y) > f_p(x^t)$: $x^t \leftarrow y$
End for
End for
$x^{\text{best}} \leftarrow \text{argmax}\left\{f_p(x^t), \forall t\right\}$

Algorithm 4: EvaluateEquilibria()
$MAXDIST$= maximum distance from existing cluster
$MINSUPPORT$= minimum support criterion to avoid local minimum
Vector with number of points per cluster $w = (w^1, \ldots, w^R) \in \mathbb{Z}^R \leftarrow 0$
$c_1 \in \mathcal{C} \leftarrow a^1$, $C \leftarrow 1$, $w^1 \leftarrow 1$
For $r = 1$ to R do
$c^{\text{closest}} \leftarrow \text{argmin}\left\{\left\|a^r - c^i\right\|^2, \forall c^i \in \mathcal{C}\right\}$
If $\left\|a^r - c^{\text{closest}}\right\|^2 < MAXDIST$ do
$c^i \leftarrow (c^i w^i + a^r)/(w^i + 1)$
$w^i \leftarrow w^i + 1$
Else $c_2 \in \mathcal{C} \leftarrow a^r$, $C \leftarrow C + 1$, $w^C \leftarrow 1$
End if
End for
For $i = 1$ to C do
If $w^i < MINSUPPORT$ do $w^i \leftarrow 0$
Else for $j = 1$ to C do
If $j \neq i$ & $w^j \geq MINSUPPORT$ do
If $(f_p(c_i) < f_p\left(c^j\right), \forall p)$ do $w^i \leftarrow 0$
End if
End for
End if
End for
$c^{\text{best}} \leftarrow \text{argmax}\{\sum_p f_p\left(c^i\right), \forall i \text{ with } w^i > 0\}$
$o \leftarrow c^{\text{best}}$
BestResponseAlgorithm(c^{best})
If $\left\|o - c^{\text{best}}\right\|^2 > MAXDIST$ print “Cluster is no equilibrium. Further investigation required.”

Parameter	Value	Parameter	Value
R	10	T	5
$BRMINREP$	10	$BRMAXREP$	30
$EMREP$	4	$LSREP$	30
α	(0.5, 2)	$conv$	5
$MAXDIST$	5	$MINSUPPORT$	2

Table A1.1 Algorithm parameters used in the experiment

Appendix 2: ANOVA output

Analysis of variance table – 3 players

Variable	Df	Mean Sq	F-value	p-value	Sign	Mean Sq	F-value	p-value	Sign
Response:		$i(a_1^{*z}\|\theta^z)$				$m(a_1^{*z}\|\theta^z)$			
γ_i	1	167.965	5575.52	$<2.2*10^{-16}$	***	13365.6	3255.1	$<2.2*10^{-16}$	***
γ_e	1	0.002	0.0725	0.7877		2057.3	501.05	$<2.2*10^{-16}$	***
e_1	1	81.291	2698.43	$<2.2*10^{-16}$	***	6482.0	1578.6	$<2.2*10^{-16}$	***
(e_2, e_3)	20	2.875	95.4308	$<2.2*10^{-16}$	***	53.7	13.083	$<2.2*10^{-16}$	***
Z	1	0.093	3.0858	0.0791	*	429.7	104.64	$<2.2*10^{-16}$	***
$\gamma_i * \gamma_e$	1	0.002	0.0649	0.7990		3.0	0.7243	0.3948	
$\gamma_i * e_1$	1	81.224	2696.20	$<2.2*10^{-16}$	***	301.7	73.473	$<2.2*10^{-16}$	***
$\gamma_i * (e_2, e_3)$	20	2.871	95.3011	$<2.2*10^{-16}$	***	24.5	5.9789	$1.3*10^{-15}$	***
$\gamma_i * Z$	1	0.091	3.0308	0.0819	*	2.9	0.7145	0.3981	
$\gamma_e * e_1$	1	0.108	3.5908	0.0583	*	1958.7	477.02	$<2.2*10^{-16}$	***
$\gamma_e * (e_2, e_3)$	20	0.034	1.1337	0.3065		6.7	1.6248	0.0395	**
$\gamma_e * Z$	1	0.000	0.0001	0.9920		15.5	3.7734	0.0522	*
$e_1 * (e_2, e_3)$	20	0.518	17.2018	$<2.2*10^{-16}$	***	90.0	21.919	$<2.2*10^{-16}$	***
$e_1 * Z$	1	0.015	0.4841	0.4867		131.0	31.915	$1.9*10^{-8}$	***
$(e_2, e_3) * Z$	20	0.011	0.3739	0.9947		4.2	1.0164	0.4384	***
Residuals	1905	0.030				4.1			

Table A2.1 ANOVA output for the actions in the first project of a Z-project pipeline for a player with experience e_1 and the opponents' experience vector (e_2, e_3) as a factor variable

Analysis of variance table – 2 players

Variable	Df	Mean Sq	F-value	p-value	Sign	Mean Sq	F-value	p-value	Sign
Response:		$i(a_1^{*z}\|\theta^z)$				$m(a_1^{*z}\|\theta^z)$			
γ_i	1	267.216	11867.2	$<2.2*10^{-16}$	***	1054.55	1510.9	$<2.2*10^{-16}$	***
γ_e	1	1.151	51.1196	$3.4*10^{-12}$	***	521.32	746.89	$<2.2*10^{-16}$	***
e_1	1	18.544	823.561	$<2.2*10^{-16}$	***	58.99	84.519	$<2.2*10^{-16}$	***
e_2	5	1.475	65.5167	$<2.2*10^{-16}$	***	178.11	255.18	$<2.2*10^{-16}$	***
Z	1	0.069	3.0677	0.0805	*	132.88	190.38	$<2.2*10^{-16}$	***
$\gamma_i * \gamma_e$	1	0.449	19.9240	$1.0*10^{-5}$	***	0.58	0.8291	0.3630	
$\gamma_i * e_1$	1	8.845	392.829	$<2.2*10^{-16}$	***	1.11	1.5936	0.2074	
$\gamma_i * e_2$	5	0.179	7.9488	$3.3*10^{-7}$	***	2.35	3.3718	0.0053	***
$\gamma_i * Z$	1	0.194	8.6360	0.0035	***	4.43	6.3520	0.0121	**
$\gamma_e * e_1$	1	0.252	11.1748	0.0001	***	173.38	248.41	$<2.2*10^{-16}$	***
$\gamma_e * e_2$	5	0.079	3.5118	0.0040	**	32.84	47.046	$<2.2*10^{-16}$	***
$\gamma_e * Z$	1	0.009	0.3921	0.5315		4.38	6.2805	0.0126	**
$e_1 * e_2$	5	0.027	1.2185	0.2992		26.29	37.666	$<2.2*10^{-16}$	***
$e_1 * Z$	1	0.589	26.1733	$4.6*10^{-7}$	***	25.34	36.305	$3.4*10^{-9}$	***
$e_2 * Z$	5	0.119	5.2837	$9.8*10^{-5}$	***	14.00	20.052	$<2.2*10^{-16}$	***
Residuals	468	0.023				0.70			

Table A2.2 ANOVA output for the actions in the first project of a Z-project pipeline for a player with experience e_1 and the opponent's experience level e_2 as a factor variable